

New systematics in charmless strange $B^+ \rightarrow VP$ decays

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Abstract

Latest data on charmless strange vector-pseudoscalar B^+ decays now including $B^+ \rightarrow \rho^+ K^0$ confirm a simple penguin model in which the gluon G in an initial $\bar{s}uG$ state fragments equally into $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ and all form factors are equal. A search for possible additional contributions shows only a few signals not obscured by experimental errors whose implications are discussed. The experimental value of 0.25 ± 0.11 for the ratio of the branching ratios $BR(B^+ \rightarrow K^{*+}\eta)$ to $BR(B^+ \rightarrow K^{*+}\eta')$ confirms the parity selection rule prediction 0.32. Large violations arise in a new sum rule for the sum of these branching ratios, analogous to the similar pseudoscalar sum rule including $K^+\eta$ and $K^+\eta'$. Indications for either an electroweak penguin contribution or additional admixtures like intrinsic charm in the $\eta - \eta'$ system remain to be clarified. An alternative symmetry description with new predictive power clarifies the simple penguin approximation and presents new predictions which can be tested experimentally. The fragmentation of the $\bar{s}uG$ state into two mesons is described by a strong interaction S-matrix dominated by nonexotic hadron resonances in multiparticle intermediate states.

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I. IMPLICATIONS OF EXPERIMENTAL SYSTEMATICS IN CHARMLESS STRANGE B^+ VECTOR-PSEUDOSCALAR DECAYS

New experimental data [1] on $B^+ \rightarrow \rho^+ K^0$ satisfy the prediction [2–4]

$$BR(B^+ \rightarrow \rho^+ K^0) = BR(B^+ \rightarrow \phi K^+) \quad (1.1)$$

The paper [1] quotes the assumption [5–7] $p'_V = -p'_P$ where p'_V and p'_P denote the amplitudes for the spectator quark to appear in the vector or pseudoscalar meson. The relation between the magnitudes

$$|p'_V| = |p'_P| \quad (1.2)$$

is actually sufficient to obtain the prediction (1.1).

This neglect of differences between vector and pseudoscalar form factors at the weak and spectator vertices appears completely unjustified in the conventional descriptions. We therefore look for an alternative approach and search for some underlying symmetry. A penguin diagram for B^+ decays into charmless strange vector-pseudoscalar states begins with a weak interaction that produces an $\bar{s}u$ gluon state. The gluon and \bar{s} are emitted in opposite directions in the rest system of the spectator quark. This finishes the weak part of the process. The gluon must then interact with both others to produce the final state. The common assumption that the gluon first produces a $q\bar{q}$ pair before interaction with the others can be questioned. We look for other descriptions that go beyond this approximation. If the strong interaction conserves flavor SU(3) symmetry, the symmetry alone places serious constraints on the observable branching ratios. These are completely independent of the detailed dynamics and approximations like factorization used in conventional approaches. We shall show that this symmetry approach leads to the relation (1.2) without any discussion of form factors.

An $\bar{s}u$ gluon state is an octet in flavor SU(3) and is a vector in the V -spin subgroup of SU(3) with $V_z = 1$. Its eigenvalue can be either even or odd under G_V parity, the analog of G parity with isospin rotated into V spin. Since G_V is conserved in QCD interactions the amplitude for the final two meson state is a linear combination of an even G_V amplitude and an odd G_V amplitude. If both amplitudes contribute to the final two-meson state the relative branching ratios will be unpredictable. The branching ratios depend upon the relative magnitude and phase of the two amplitudes which are determined by unknown QCD dynamics. If only the odd G_V amplitude contributes, the relation between (1.2) immediately follows. The odd G_V amplitude has the flavor quantum numbers of the kaon and leads to branching ratios determined from V spin like those from decay of a high mass kaon.

The search for a justification for neglecting the other even G_V amplitude is discussed in detail below. It follows automatically if the QCD strong interaction S-matrix is dominated by S channel resonances which all have odd G_V parity.

We now examine the systematics of the experimental data and then discuss the symmetry picture in detail including new predictions for other final states.

A. Experimental tests of the simplest extreme penguin model

We first examine the simplest gluonic penguin model in which the gluon G in an initial $\bar{s}uG$ states fragments equally into $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$, all form factors are equal and the OZI rule [8–12] is respected; i.e. the $q\bar{q}$ pair produced by the gluon does not end in the same final meson. This gives the relations

$$\begin{aligned}
2BR(B^+ \rightarrow K^+\omega) &= 2BR(B^+ \rightarrow K^+\rho^0) = BR(B^+ \rightarrow K^0\rho^+) = \\
&= 2BR(B^+ \rightarrow K^{*+}\pi^0) = BR(B^+ \rightarrow K^{*0}\pi^+) = BR(B^+ \rightarrow \phi K^+) \\
\text{Data} &\quad \text{From} &\quad \text{BaBar} \\
12.2 \pm 1.2 \pm 0.8 &= 10.2 \pm 1.6 \pm 1.6 &= 8.0 \pm 1.4 \pm 0.5 = \\
= 13.8 \pm 4.0 \pm 2.6 &= 13.5 \pm 1.2 \pm 0.9 &= 8.4 \pm 0.7 \pm 0.7 \\
\text{Data} &\quad \text{From} &\quad \text{HFAG - Avg} \\
13.6 \pm 1.0 &= 8.5 \pm 1.1 &= 8.0 \pm 1.5 = \\
= 13.8 \pm 4.6 &= 10.7 \pm 0.8 &= 8.3 \pm 0.65
\end{aligned} \tag{1.3}$$

where we have used the data from BaBar quoted by HFAG [13] and also the HFAG Average listed below in units of 10^{-6} and do not include the final states including the η and η' . This is still reasonably good and not sufficiently precise to pinpoint other contributions omitted in this simple picture. The one exception is the large $BR(B^+ \rightarrow K^+\omega)$ seen in the HFAG average although not in the BaBar data.

Each of the two individual lines of equalities in eq.(1.3) is independent of the form factor assumption, (1.2). The first line gives equalities between transitions to final states where the vector meson contains the spectator quark; the second gives equalities between transitions to final states where the vector meson contains the \bar{s} antiquark from the weak vertex. The assumption of equal form factors (1.2) is needed only to relate the two lines. The data and in particular the relation (1.1) confirm the equality between the two lines and therefore the assumption (1.2).

The relations for the neutral decays corresponding to (1.3) for the charged decays are:

$$\begin{aligned}
2BR(B^0 \rightarrow K^0\omega) &= 2BR(B^0 \rightarrow K^0\rho^0) = BR(B^0 \rightarrow K^+\rho^-) = \\
&= 2BR(B^0 \rightarrow K^{*0}\pi^0) = BR(B^0 \rightarrow K^{*+}\pi^-) = BR(B^0 \rightarrow \phi K^0) \\
\text{Data} &\quad \text{From} &\quad \text{HFAG - Avg} \\
10.4 \pm 1.4 &= 10.8 \pm 2.0 &= 15.3 \pm 3.7 \\
= 0.0 \pm 2.6 &= 9.8 \pm 1.1 &= 8.3 \pm 1.2
\end{aligned} \tag{1.4}$$

These are also in reasonable agreement except for the small $B^0 \rightarrow K^{*0}\pi^0$ decay.

The relations (1.3) are also obtainable by noting that initial state has the flavor and parity quantum numbers of a kaon and using SU(3) flavor symmetry to relate the decays of a high mass kaon.

B. Experimental data used in our analyses

We have used the BaBar experimental data [13] and also the HFAG Average listed below in units of 10^{-6} and leave the combination of statistical and systematic errors for the reader.

Transition	Babar Data	HFAG – Avg	Momentum	
$BR(B^+ \rightarrow K^+\omega)$	$= 6.1 \pm 0.6 \pm 0.4$	$= 6.8 \pm 0.5;$	$p = 2557$	
$BR(B^+ \rightarrow K^+\rho^0)$	$= 5.1 \pm 0.8 \pm 0.8$	$= 4.25 \pm 0.56;$	$p = 2558$	
$BR(B^+ \rightarrow K^0\rho^+)$	$= 8.0 \pm 1.4 \pm 0.5$	$= 8.0 \pm 1.5;$	$p = 2558$	
$BR(B^+ \rightarrow K^{*+}\eta)$	$= 18.9 \pm 1.8 \pm 1.3$	$= 19.3 \pm 1.6;$	$p = 2534$	(1.5)
$BR(B^+ \rightarrow K^{*+}\eta')$	$= 4.9 \pm 1.9 \pm 0.8$	$= 4.9 \pm 2.1;$	$p = 2472$	
$BR(B^+ \rightarrow K^{*0}\pi^+)$	$= 13.5 \pm 1.2 \pm 0.9$	$= 10.7 \pm 0.8;$	$p = 2562$	
$BR(B^+ \rightarrow K^{*+}\pi^0)$	$= 6.9 \pm 2.0 \pm 1.3$	$= 6.9 \pm 2.3;$	$p = 2562$	
$BR(B^+ \rightarrow \phi K^+)$	$= 8.4 \pm 0.7 \pm 0.7$	$= 8.3 \pm 0.65;$	$p = 2516$	

Transition	Babar Data	HFAG – Avg	Momentum	
$BR(B^0 \rightarrow K^0\omega)$	$= 6.2 \pm 1.0 \pm 0.4$	$= 5.2 \pm 0.7;$	$p = 2557$	
$BR(B^0 \rightarrow K^0\rho^0)$	$= 4.9 \pm 0.8 \pm 0.9$	$= 5.4 \pm 1.0;$	$p = 2558$	
$BR(B^0 \rightarrow K^+\rho^-)$	$=$	$= 15.3 \pm 3.7;$	$p = 2558$	
$BR(B^0 \rightarrow K^{*0}\eta)$	$= 16.5 \pm 1.1 \pm 0.8$	$= 15.9 \pm 1;$	$p = 2534$	(1.6)
$BR(B^0 \rightarrow K^{*0}\eta')$	$= 3.8 \pm 1.1 \pm 0.5$	$= 3.8 \pm 1.2;$	$p = 2472$	
$BR(B^0 \rightarrow K^{*0}\pi^0)$	$=$	$= 0.0 \pm 1.3;$	$p = 2562$	
$BR(B^0 \rightarrow K^{*+}\pi^-)$	$= 11.0 \pm 0.4 \pm 0.7$	$= 9.8 \pm 1.1;$	$p = 2562$	
$BR(B^0 \rightarrow \phi K^0)$	$= 8.4 \pm 1.5 \pm 0.5$	$= 8.3 \pm 1.2;$	$p = 2516$	

C. Possible signals from violations of the simplest gluonic penguin model

Our approach here is complementary to the extensive analysis [6] which uses a more detailed model with more parameters to fit much more data. We concentrate here on the simple penguin model which seems to do too well and look for the inevitable signals for its breakdown which are mainly still obscured by experimental errors. We include updated data for $B^0 \rightarrow K^{*0}\pi^0$, $B^0 \rightarrow K^0\rho^0$ and $B^+ \rightarrow K^0\rho^+$ not yet available to ref. [6] and pointed out there as “soon to be seen”.

We first note that the penguin diagram produces an isospin eigenstate with $I = 1/2$. Thus the transtions to the two $K^*\pi$ and the two $K\rho$ final states are related by isospin and completely independent of all form factors. The factors of 2 appearing in eqs. (1.3) and (1.4) are isospin Clebsch-Gordan coefficients. Any violation of these isospin equalities indicates either an isospin violation, as in an electroweak penguin contribution, or an isospin $I = 3/2$ contribution, as in a tree diagram contribution. The two violations of the simple gluonic penguin that we have noted above, the large $BR(B^+ \rightarrow K^+\omega)$ and the small $BR(B^0 \rightarrow K^{*0}\pi^0)$ have been disucssed in ref. [6] and can be due to electroweak penguin contributions.

The relation

$$\frac{BR(B^+ \rightarrow K^+\omega)}{BR(B^+ \rightarrow K^+\rho^0)} = 1 \quad (1.7)$$

follows from any combination of penguin and tree amplitudes [4] but can be broken by an electroweak penguin [6]. The experimental data in ref. [6] gave 1.3 ± 0.3 consistent with 1. The new data violate the relation (1.7) and may indicate an EWP contribution. The small $BR(B^0 \rightarrow K^{*0}\pi^0)$ is predicted in ref. [6] as due to the EWP contribution. Their prediction of 1×10^{-6} is still consistent with the smaller value in the new data.

D. A good relation between the ratio of $BR(B^+ \rightarrow K^{*+}\eta')$ to $BR(B^+ \rightarrow K^{*+}\eta)$

The high mass kaon model also predicts the observed suppression of $BR(B^+ \rightarrow K^{*+}\eta')$ relative to $BR(B^+ \rightarrow K^{*+}\eta)$. A high mass K^+ goes into a high mass π^+ under the U -spin reflection $s \leftrightarrow d$. The decay $\pi^+ \rightarrow \rho^+\pi^0$ is allowed by G parity; the decay $\pi^+ \rightarrow \rho^+\eta$ is forbidden. U -spin reflections of these decays change:

$$\pi^+ \leftrightarrow K^+; \quad \rho^+ \leftrightarrow K^{*+}; \quad G \leftrightarrow G_V; \quad \eta_u \pm \eta_d \leftrightarrow \eta_u \pm \eta_s \quad (1.8)$$

where G_V parity is defined with the V -spin (us) subgroup of flavor $SU(3)$ like the ordinary G parity is defined with isospin and the pseudoscalar flavor states $|\eta_i\rangle$ are defined as pseudoscalars created from a $q\bar{q}$ pair with flavors i that can be u , d or s and the nonstrange pseudoscalars are

$$|\pi^0\rangle \equiv \frac{|\eta_u\rangle - |\eta_d\rangle}{\sqrt{2}}; \quad |\eta_n\rangle \equiv \frac{|\eta_u\rangle + |\eta_d\rangle}{\sqrt{2}} \quad (1.9)$$

These show that the decay $K^+ \rightarrow K^{*+}(\eta_u - \eta_s)$ is allowed by G_V parity; while the decay $K^+ \rightarrow K^{*+}(\eta_u + \eta_s)$ is forbidden. Since the η and η' wave functions in the standard mixing model combines the η_u and η_s components with a positive phase in the η' and a negative phase in the η , the transition matrices in the $\eta - \eta'$ basis are related as

$$\frac{\langle K^{*+}\eta' | T | B^+ \rangle}{\langle K^{*+}\eta | T | B^+ \rangle} = \frac{\langle \eta' | \eta_u - \eta_s \rangle}{\langle \eta | \eta_u - \eta_s \rangle} = -\frac{1}{\sqrt{8}} = \frac{\langle \eta' | \eta_d - \eta_s \rangle}{\langle \eta | \eta_d - \eta_s \rangle} = \frac{\langle K^{*0}\eta' | T | B^+ \rangle}{\langle K^{*0}\eta | T | B^+ \rangle} \quad (1.10)$$

where we have used isospin and the fact the the penguin final state is isoscalar to include the neutral decays and substituted the mixing angle commonly used [14]

$$|\eta\rangle = \frac{|\eta_u\rangle + |\eta_d\rangle - |\eta_s\rangle}{\sqrt{3}}; \quad |\eta'\rangle = \frac{|\eta_u\rangle + |\eta_d\rangle + 2|\eta_s\rangle}{\sqrt{6}} \quad (1.11)$$

This qualitatively predicts the observed suppression of $BR(B^+ \rightarrow K^{*+}\eta')$ relative to $BR(B^+ \rightarrow K^{*+}\eta)$ and notes correctly that the suppression factor for the vector-pseudoscalar case is much less than the infinite suppression for the two pseudoscalar case predicted by this mixing (1.13).

$$\frac{\langle K^+\eta | T | B^+ \rangle}{\langle K^+\eta' | T | B^+ \rangle} = \frac{\langle \eta | \eta_u + \eta_s \rangle}{\langle \eta' | \eta_u + \eta_s \rangle} = 0 \quad (1.12)$$

For a better approximation we use a general mixing angle

$$|\eta'\rangle = |\eta_n\rangle \cos \theta + |\eta_s\rangle \sin \theta; \quad |\eta\rangle = |\eta_n\rangle \sin \theta - |\eta_s\rangle \cos \theta \quad (1.13)$$

Then

$$\frac{\langle K^+\eta | T | B^+ \rangle}{\langle K^+\eta' | T | B^+ \rangle} = \frac{\langle \eta | \eta_u + \eta_s \rangle}{\langle \eta' | \eta_u + \eta_s \rangle} = \frac{\sin \theta - \sqrt{2} \cos \theta}{\cos \theta + \sqrt{2} \sin \theta} = \frac{\tan \theta - \sqrt{2}}{1 + \sqrt{2} \tan \theta} \quad (1.14)$$

Solving for the mixing angle θ in terms of the experimentally measured ratio denoted by X gives

$$\tan \theta = \frac{\sqrt{2} + X}{1 - X\sqrt{2}}; \quad X \equiv \frac{\langle K^+ \eta | T | B^+ \rangle}{\langle K^+ \eta' | T | B^+ \rangle} \approx \sqrt{\left[\frac{2.2 \pm 0.3}{69.7 \pm 3.8} \right]} = .178 \quad (1.15)$$

This then predicts

$$\frac{\langle K^{*+} \eta' | T | B^+ \rangle}{\langle K^{*+} \eta | T | B^+ \rangle} = \frac{\langle \eta' | \eta_u - \eta_s \rangle}{\langle \eta | \eta_u - \eta_s \rangle} = \frac{1 - \sqrt{2} \tan \theta}{\tan \theta + \sqrt{2}} = \frac{1 + 2X\sqrt{2}}{X - 2\sqrt{2}} = -.567 \quad (1.16)$$

or

$$\frac{BR(B^+ \rightarrow K^{*+} \eta')}{BR(B^+ \rightarrow K^{*+} \eta)} = \frac{BR(B^o \rightarrow K^{*o} \eta')}{BR(B^o \rightarrow K^{*o} \eta)} \approx \left| \frac{\langle K^{*+} \eta' | T | B^+ \rangle}{\langle K^{*+} \eta | T | B^+ \rangle} \right|^2 = 0.32 \quad (1.17)$$

The experimental values are in good agreement with this prediction

$$\frac{BR(B^+ \rightarrow K^{*+} \eta')}{BR(B^+ \rightarrow K^{*+} \eta)} = \frac{4.9 \pm 2.1}{19.3 \pm 1.6} = 0.25 \pm 0.11 \quad (1.18)$$

$$\frac{BR(B^o \rightarrow K^{*o} \eta')}{BR(B^o \rightarrow K^{*o} \eta)} = \frac{3.8 \pm 1.2}{15.9 \pm 1} = 0.24 \pm 0.08 \quad (1.19)$$

E. A bad relation for the sum $BR(B^+ \rightarrow K^{*+} \eta') + BR(B^+ \rightarrow K^{*+} \eta)$ independent of form factor differences

We now examine more carefully the decays to the final states $K^{*+} \eta$ and $K^{*+} \eta'$ where significant violations of the simplest model occur. To pinpoint these violations we choose relations that are independent of form factor differences and do not depend on the relation (1.2). We assume that the penguin contribution to B decays into two charmless strange mesons denoted by M_1 and M_2 goes via the transition

$$|B^+\rangle \rightarrow |(\bar{s}q_g)_W; (\bar{q}_g u)_S\rangle \rightarrow |M_1; M_2\rangle \quad (1.20)$$

where q_g and \bar{q}_g denote a quark of pair of any flavor. The subscript g denotes that they come from the same gluon. The subscript W denotes that the pair contains a \bar{s} antiquark produced at the weak vertex and the subscript S denotes that the pair contains the spectator u quark. We assume that the transition is flavor independent but the hadronization form factors for weak and spectator vertices can be different. The transition matrix element therefore satisfies the relation,

$$\langle (\bar{s}u_g)_W; (\bar{u}_g u)_S | T_P | B^+ \rangle = \langle (\bar{s}d_g)_W; (\bar{d}_g u)_S | T_P | B^+ \rangle = \langle (\bar{s}s_g)_W; (\bar{s}_g u)_S | T_P | B^+ \rangle \quad (1.21)$$

Since the spectator u quark remains in all transitions, the transition that would require a spectator d quark vanishes. Thus the states $K^{*+} \eta_n$ and $K^{*+} \pi^o$ are both produced equally via the $K^{*+}(u\bar{u})$ state

$$BR(B^+ \rightarrow K^{*+} \eta_n) = BR(B^+ \rightarrow K^{*+} \pi^o) \quad (1.22)$$

where we neglect phase space corrections.

The $B^+ \rightarrow K^{*+}\eta_s$ and $B^+ \rightarrow K^o\rho^+$ transitions both have final states with a vector meson containing the spectator quark and a pseudoscalar containing the strange antiquark from the weak $\bar{b} \rightarrow \bar{s}$ transitions. The form factors are the same and

$$BR(B^+ \rightarrow K^{*+}\eta_s) = BR(B^+ \rightarrow K^o\rho^+) \quad (1.23)$$

Combining equations (1.22) and (1.23) gives a sum rule independent of the standard model mixing angle which can be compared with experiment.

$$\begin{aligned} BR(B^+ \rightarrow K^{*+}\eta_n) + BR(B^+ \rightarrow K^{*+}\eta_s) &= BR(B^+ \rightarrow K^o\rho^+) + BR(B^+ \rightarrow K^{*+}\pi^o) \\ BR(B^+ \rightarrow K^{*+}\eta) + BR(B^+ \rightarrow K^{*+}\eta') &= BR(B^+ \rightarrow K^o\rho^+) + BR(B^+ \rightarrow K^{*+}\pi^o) \\ \text{BaBar Data} \\ (18.9 \pm 1.8 \pm 1.3) + (4.9 \pm 1.9 \pm 1.8) = 23.8 \pm X &= (8.0 \pm 1.4 \pm 0.5) + (6.9 \pm 2.0 \pm 1.3) = 14.9 \pm X \\ \text{FFAG - Avg} \\ (19.3 \pm 1.6) + (4.9 \pm 2.1) = 24.2 \pm X &= (8.0 \pm 1.5) + (6.9 \pm 2.3) = 14.9 \pm X \end{aligned} \quad (1.24)$$

This sum rule is seen to be seriously violated in the same way as the previous pseudoscalar sum rule [2] indicating an additional contribution to the $\eta - \eta'$ system.

$$BR(B^\pm \rightarrow K^\pm\eta') + BR(B^\pm \rightarrow K^\pm\eta) \leq BR(B^\pm \rightarrow K^\pm\pi^o) + BR(B^\pm \rightarrow \tilde{K}^o\pi^\pm) \quad (1.25)$$

where \tilde{K}^o denotes K^o for the B^+ decay and \bar{K}^o for the B^- decay. The experimental values [13] in units of 10^{-6} are

$$BR(K^\pm\eta')(69.7 \pm 3.8) + BR(K^\pm\eta)(2.2 \pm 0.3) \leq BR(K^\pm\pi^o)(12.8 \pm 0.6) + BR(K^o\pi^\pm)(23.1 \pm 1.0) \quad (1.26)$$

Whether this additional contribution arises from an electroweak penguin contribution [6] or a difference in the wave functions from the standard quark model is an open challenge for QCD. The possibility of adding “intrinsic charm” to the wave functions [15–18] would mix an η_c into the η and/or η' wave function.

The branching ratio $BR(B^+ \rightarrow K^+\eta_c)$ is $9.1 \pm 1.3 \times 10^{-4}$ which is larger by a factor of 38 than the charmless branching ratio $BR(B^+ \rightarrow K^o\pi^+)$, 24.1×10^{-6} . The difference in phase space indicates an even larger ratio of the squares of the transition matrix elements. Thus an admixture of only a few per cent of η_c into the η and/or η' wave function could eliminate the disagreements with the sum rules. It would be of interest to find an experimental test which would distinguish between an electroweak penguin contribution and between intrinsic charm or η_c admixture in the wave functions. Most tests in standard $\eta - \eta'$ spectroscopy are not sensitive enough to detect these small admixtures, but EWP contributions may be related to other observables.

II. A SYMMETRY APPROACH TO CHARMLESS STRANGE B^+ VECTOR-PSEUDOSCALAR DECAYS

A. Exotic and nonexotic final states

We now search for an alternative symmetry approach that can distinguish between relations that neglect and exhibit the form factor differences.

The penguin diagram for the decay of a pseudoscalar B^+ meson to a charmless strange vector pseudoscalar state has a final quark-antiquark-gluon state of odd parity with the flavor quantum numbers of a kaon. It is the strange member of a pseudoscalar SU(3) flavor octet but has two possible eigenvalues for the generalized charge conjugation operator, also known as “isoparity”, which defines the relative phases of the charge conjugate states in the same octet and the eigenvalue under charge conjugation of its C-eigenstate members. Dothan [19] generalized the idea of G parity from SU(2) to SU(3). This developed further [20] on generalizations of isoparity. For weak interactions, the generalizations of G parity from SU(2) to SU(n) is not directly relevant because charge conjugation is badly violated like parity in weak interactions. However one can multiply G parity by normal space-inversion parity to make GP.

We call the generalization of GP to SU(3) and SU(n) “Dothan parity” and investigate its relevance to weak decays. We first consider the V- spin (us) subgroup of flavor SU(3) and the G_V parity. The ordinary K^+ charged strange pseudoscalar meson is a member of an octet whose charge-conjugate state, the π^0 is even under C. The pion isotriplet has isospin one and odd G parity. The K^+ has V spin 1 and odd G_V parity. An “exotic” pseudoscalar octet can be defined whose nonstrange isovector is odd under charge conjugation and whose positively charged strange member has V spin 1 and even G_V parity. This state is called exotic because it cannot be made from a quark-antiquark pair. However, it can be made from a quark-antiquark pair and a gluon and can a priori occur in a penguin diagram. Since the parity of the state is negative, the normal state has even $G_V P$ like the pion has even GP ; the exotic state has odd $G_V P$ and its nonstrange isovector has odd GP opposite to that of the pion.

Since the relations (1.3) are obtainable by using SU(3) flavor symmetry to relate the decays of a high mass kaon, we see that these relations assume that the initial state is “normal” and that contributions from an exotic initial state are neglected. Since GP is simply related to CP, one can look for a possibility of identifying the normal state with a CP-conserving transition and the exotic state with a CP-violating transition.

For a simple case we first examine the $B_s \rightarrow \phi\eta$ decay. The $B_s(\bar{b}s)$ is a pseudoscalar meson with odd parity. If it is a member of any symmetry multiplet which includes $\bar{b}b$ and $\bar{s}s$ pseudoscalar states; e.g. an SU(2) symmetry in the bs flavor space, the multiplet is even under charge conjugation and therefore odd under CP. The $\phi\eta$ p-wave state which can be produced in $B_s(\bar{b}s)$ decay is also odd under parity but odd under charge conjugation and even under CP. Thus if this generalized CP is conserved the decay $B_s \rightarrow \phi\eta$ is forbidden. Any symmetry which contains both the $\bar{b}b$ and $\bar{s}s$ pseudoscalar states is badly broken by mass differences. However we first consider the implications of such symmetries for weak decays and leave symmetry breaking for later analysis.

We note that the final states $K^{*+}K^-$ and $K^{*-}K^+$ go into one another under CP and

similarly for $K^{*o}\bar{K}^o$ and $\bar{K}^{*o}K^o$. Thus conservation of this generalized CP predicts

$$\begin{aligned} BR(B_s \rightarrow \phi\eta) &= BR(B_s \rightarrow \phi\eta') = 0 \\ BR(B_s \rightarrow K^{*o}\bar{K}^o) &= BR(B_s \rightarrow \bar{K}^{*o}K^o) \\ BR(B_s \rightarrow K^{*+}K^-) &= BR(B_s \rightarrow K^{*-}K^+) \end{aligned} \quad (2.1)$$

Note that in the dominant penguin diagram

$$B_s(\bar{b}s) \rightarrow (\bar{s}Gs) \rightarrow (\bar{s}u)(\bar{u}s) \rightarrow K^+K^- \quad (2.2)$$

where K here denotes either pseudoscalar or vector meson, the positive kaon is produced by combining the u with the \bar{s} produced in the weak interaction; the negative kaon is produced by combining the \bar{u} with the spectator s quark. The form factors for producing vector and pseudoscalar kaons from the two vertices are not expected to be equal. So the equality and the selection rules are nontrivial and their conformation or violation is interesting.

We now consider a (ubs) SU(3) flavor symmetry which is the analog of the usual (uds) symmetry with the d replaced by the b . Since there are no d quarks in the B^+ or K^+ we can consider the weak decay

$$B^+ \rightarrow K^+G \quad (2.3)$$

as a transition between two octet states in the (ubs) classification. We can also define the phases of the CKM matrix for the weak interaction in the standard model with all CKM phases real in the (ubs) flavor subspace. There is therefore no CP violation in this flavor subspace.

If we now combine CP with flavor in this subspace to make Dothan parity, we find that neutral members of the flavor octets on both sides of eq. (2.3) which are eigenstates of CP must have the same eigenvalue. This tells us that the K^+G state must have the same G_V parity as the kaon and that the exotic state is forbidden.

The relevant SU(3) coupling in conventional (uds) SU(3) is a (VPK) coupling of three octets which is unique since the strong interaction conserves charge conjugation. It is therefore also the same as the $K\pi$, $K\eta$ and $K\eta'$ couplings to the $K^*(890)$.

The experimental consequences of the extreme assumption (1.3) is equivalent on one hand to the neglect of the differences between vector and pseudoscalar form factors or on the other hand is equivalent to considering only SU(3) octet final states with normal generalized charge conjugation and neglecting the exotic contribution.

B. Possible dynamical justification for neglecting exotic final states

Most conventional calculations of penguin diagrams do not consider the classification of final states as exotic nor nonexotic. To define more precisely the differences between the two approaches we first write a general expression for the transition matrix element of a penguin diagram. The charmless strange B^+ decay is a weak transition denoted by an operator T_W to an intermediate state $|\bar{s}uG\rangle$ followed by a strong transition denoted by an operator T_S to the final state.

$$\langle f | T_P | B^+ \rangle = \langle f | T_S | \bar{s}uG \rangle \cdot \langle \bar{s}uG | T_W | B^+ \rangle \quad (2.4)$$

The strong transition can be written as

$$\langle f | T_S | \bar{s}uG \rangle = \sum_i \langle f | H_i^{norm} \rangle \cdot \langle H_i^{norm} | T_S | \bar{s}uG \rangle + \sum_i \langle f | H_i^{ex} \rangle \cdot \langle H_i^{ex} | T_S | \bar{s}uG \rangle \quad (2.5)$$

where the sum over a complete set of strong interaction eigenstates is separated into a sum over “normal” states having odd G_V denoted by $|H_i^{norm}\rangle$ and a sum over exotic states having even G_V denoted by $|H_i^{ex}\rangle$. Since G_V is conserved by strong interactions that conserve flavor SU(3), the two sums are independent and there is no mixing between the states in the two summations.

We now see that if the strong interaction conserves flavor SU(3) symmetry, the symmetry alone places serious constraints on the observable branching ratios. These are completely independent of detailed dynamical assumptions like heavy quark symmetry, helicity conservation or factorization.

The final two meson state is a linear combination of the contributions from the two summations in eq. (2.5), the even G_V summation and the odd G_V summation. The branching ratios depend upon the relative magnitude and phase of the two summations which are determined by unknown QCD dynamics. A very small QCD interaction normally neglected will act differently on the two summations if they are both appreciable and destroy any relative magnitude and phase coherence. Assuming that only the odd G_V summation contributes leads to our results where the amplitude with the quantum numbers of the kaon gives branching ratios determined from V spin like those from decay of a high mass kaon.

Feasible calculations involve choosing a particular set of intermediate states in eq. (2.5) and neglecting the contributions of others. Most common calculations; e.g. refs. [5–7] consider only states of two quark-antiquark pairs and neglect multiparticle intermediate states. The symmetry approach considered here neglects all intermediate states $|H_i^{ex}\rangle$ having exotic flavor quantum numbers but includes all multiparticle states having normal quantum numbers. This corresponds to the dual-resonance-model [23] approach in which the strong interaction S matrix is represented by the sum of S channel resonances which all have non-exotic quantum numbers.

It is difficult at this point to decide which approach is confirmed by rigorous QCD, neglect of multiparticle intermediate states or neglect of exotic intermediate states. However, the calculation considering only states of two quark-antiquark pairs generally do not analyze the G_V parity of their expressions. If a G_V analysis of their final states involve appreciable contributions having both G_V parities, their predictions can be destroyed by any neglected small QCD interaction.

QCD factorization is natural in the standard tree diagram where a \bar{b} antiquark emits a high momentum W which leaves the spectator quark before it hadronizes into a meson and has no further strong interactions. The penguin diagram is very different as the gluon and \bar{s} are emitted in opposite directions in the rest system of the spectator quark. The gluon must then interact with both others to produce the final state. The assumption that the gluon first produces a $q\bar{q}$ pair before interaction with the others can be questioned. At this stage one can compare the experimental consequences of both approaches.

III. FURTHER EXPERIMENTAL TESTS OF THE NONEXOTIC MODEL

We now examine further experimental tests of the nonexotic model. The assumption that charmless strange B decays are dominated by transitions via intermediate states with nonexotic quantum numbers can immediately be tested in all final states. Only even values of $G_V P$ contribute; i.e. odd G_V for odd parity final states and even G_V for even parity states. This immediately leads to the analog for vector-vector final states of the prediction (1.1), which is seen to agree with experiment.

$$\begin{aligned} BR(B^+ \rightarrow \rho^+ K^{*o}) &= BR(B^+ \rightarrow \phi K^{*o}) \\ &= 9.2 \pm 1.5 \qquad \qquad = 9.7 \pm 1.5 \end{aligned} \quad (3.1)$$

where we have used the data from the HFAG Average [13]. Further data will be able to test the nonexotic model by the relations from the full generalization to the other cases of the vector pseudoscalar relations (1.3). For the vector-vector case,

$$\begin{aligned} 2BR(B^+ \rightarrow K^{*+} \omega) &= 2BR(B^+ \rightarrow K^{*+} \rho^0) = BR(B^+ \rightarrow K^{*o} \rho^+) = BR(B^+ \rightarrow \phi K^+) \\ &= \qquad \qquad \qquad = 9.2 \pm 1.5 \qquad \qquad = 9.7 \pm 1.5 \end{aligned} \quad (3.2)$$

Analogous predictions can be made for other final states; e.g. the tensor- pseudoscalar case including the $K_2^*(1430)$ tensor resonance which also has even parity and will be dominated by even G_V states. There is also the prediction for the TP final states including the η and η' .

$$\frac{BR(B^+ \rightarrow K_2^{*+} \eta)}{BR(B^+ \rightarrow K_2^{*+} \eta')} = \frac{BR(B^0 \rightarrow K_2^{*o} \eta)}{BR(B^0 \rightarrow K_2^{*o} \eta')} = \frac{BR(B^+ \rightarrow K^+ \eta)}{BR(B^+ \rightarrow K^+ \eta')} = \frac{BR(B^0 \rightarrow K^o \eta)}{BR(B^0 \rightarrow K^o \eta')} \quad (3.3)$$

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